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Roll No. : ....

MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY

NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 U.P.

Sessional Examination/Class Test – 1 (Odd Semester, 2022-23)

Course/Branch: B Tech (CSE/IT/CS-IT/DS/IOT/AI/AIML) **Semester:** 3rd

Subject Name: Discrete Mathematics & Theory of Logic **Max. Marks:** 60

Subject Code: KCS - 303 **Time Allowed:** 120 min

CO-1: *On completion of this course, the student will be able to describe and apply concepts related to sets, functions, and relations, and methods of proof in the study of other discrete structures.*
CO-2: *On completion of this course, the student will be able to describe concepts needed to apply basic properties of fundamental algebraic structures.*

Section – A (CO - 1)

NOTE: Attempt **ALL** questions. (30 Marks)

1. Attempt any **six parts** of this question. Each part is for two marks. (2 x 6 = 12 Marks)

a) Find the union and intersection of the multisets A and B given by
$$A = [1, 1,4,2,2,3] \quad \text{and} \quad B = [1, 2, 2, 6, 3, 3]$$
b) Show that, for any three nonempty sets A, B, and C, we have $(A - C) \cap (C - B) = \emptyset$.
c) Let $A = \{2, 4, 5, 7, 8\}$, and R be the relation on A given by
$$R = \{(a, b) \in A \times A |\, a + b \leq 11\}.$$
Write the relation matrix of R.
d) Define reflexive closure and symmetric closure of a relation R on a set A.
e) Explain how equivalence relation on a set A is related to the partition of A.
f) Let the two functions $f, g: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 3x^2 + 2$ and $g(x) = 7x - 5$. Find the compositions fog and gof.
g) Describe the method of proof by contradiction.

2. Attempt any **three parts** of this question. Each part is for 6 marks. (3 x 6 = 18 Marks)

a) Let A, B, and C be three nonempty sets such that we have
$$A \cap C = B \cap C \quad \text{and} \quad A \cup C = B \cup C$$
Show that $A = B$.
b) In each of the following cases, given an example of a relation R on the set $A = \{a, b, c, d, e\}$:
(i) R is symmetric and antisymmetric;
(ii) R is neither symmetric nor antisymmetric;
(iii) R is irreflexive and antisymmetric.
c) Let $A = \{1,2,3,4,5,6\}$, and consider the relation
$$R = \{(1,2), (1,3), (2,4), (5,6)\}.$$
Find the transitive closures of the relation R.
d) Show that the function $f: \mathbb{N} \to \mathbb{Z}$ given by

$$f(n) = \begin{cases} \frac{n}{2}, & if\ n\ is\ even \\ \frac{1-n}{2}, & if\ n\ is\ odd \end{cases}$$

is a bijective function. Also, compute the inverse function of the function f.

e) Show that

$$3 + 33 + \cdots + 33\cdots3\ (n\ times) = \frac{10^{n+1} - 9n - 10}{27},$$

for all $n \geq 1$.

Section – B (CO - 2)

NOTE: Attempt **ALL** questions. (30 Marks)

3. Attempt any **six parts** of the question. Each part is for two marks. (2 x 6 = 12 Marks)

a) On the set of integers $\mathbb{Z}$, consider the binary operation * given by

$$a * b = a + b - 2, \text{ for } a, b \in \mathbb{Z}.$$

Find the identity element of $\mathbb{Z}$ with respect to the operation *.

b) Let $(G,*)$ be a group. Prove that $(a * b)^{-1} = b^{-1} * a^{-1}$, for all $a, b \in G$.

c) Describe the construction of a cyclic group of order 4.

d) Give an example of a group G with two subgroups H and K such that $H \cup K$ is not a subgroup.

e) Consider the permutations $a = (1\ 2\ 3)(4\ 5), b = (4\ 5)$ and $c = (1\ 5\ 2\ 4)$ of the set $A = \{1, 2, 3, 4, 5\}$. Compute the two permutations abc and bca.

f) Describe left and right cosets of a subgroup H in a group G.

g) Explain why $N = \{e, (23)\}$ is *not* a normal subgroup of the symmetric group

$$S_3 = \{e, (132), (123), (23), (13), (12)\}.$$

4. Attempt any **three parts** of the question. Each part is for 6 marks. (3 x 6 = 18 Marks)

a) Show that the set of congruence classes $\mathbb{Z}_6$ is a group with respect to operation $+_6$.

b) Let S_3 be the set of permutations of the set $\{1, 2, 3\}$. Show that (S_3, o) is a group, where o denotes the *composition of functions*.

c) Let H be a subgroup of a group $(G,*)$, and $x \in G$. Show that $x \in H$ if and only if $x * H = H$.

d) Compute cosets of the subgroup $H = 6\mathbb{Z}$ in the group $G = (\mathbb{Z}, +)$. What is the index [G: H]?

e) State the Lagrange Theorem, and hence prove that every group of prime order is cyclic.